

Specifying the Methods to Calculate Thermal Efficiency of a Dual Production Well System “Fluid-Geoheat”

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Abstract

Methods to calculate thermal efficiency of dual production well system “fluid-geoheat” have been specified for conditionally thin productive fluid-saturated seam in terms of radial planar filtration. It has been demonstrated that relative to a variant, ignoring Joule-Thompson effect, and variant of discrete Joule-Thomson coefficient substitution, calculation accuracy as for the well heat efficiency increases by 33% and 9% respectively. It has been proved that it is expedient for a conditionally thin seam to use effective temperature within the heat-exchange equitation with Fourier substitution as related to heat transfer while considering heat input in terms of unidirectional input to a virtual disk plate from the seam floor. In the context of the proposed calculation methods, increase in numerical estimations of thermal efficiency of a well results from the consideration of extra heat pumping owing to Joule-Thompson effect by rocks adjoining the productive formation.

Keywords: Hydrocarbon well; Thermal efficiency of a system; Mechanocaloric effect; Thermal diffusion; Geothermal energy production.

1. Introduction

Dual well “fluid-geoheat” mining system provides simultaneous production of hydrocarbon fluid (i.e. crude oil and natural gas) as well as geothermal energy increasing the overall well efficiency. Thus, it is important to specify methods to calculate the overall efficiency as well as thermal efficiency of such dual well systems separately. Among other things, active interest is taken in the development of thermal efficiency dependence upon the fluid temperature and bottomhole pressure [1-5]. Analysis of studies proves that their stage one should involve transition from theoretical modeling to experimental models with adequate simplifications and relevant accuracy [1-5]. The abovementioned will help obtain early assessments of thermal efficiency and mass discharge within the boundary coordinates of the developed analytical model with the selected initial conditions.

General system of integral and differential equations, describing processes in a reservoir with adjoining rocks (if thermal exchange, filtration, thermal diffusion, and viscous friction are available), involve classical equations of Leonard Euler, Claude Navier, George Stokes, Antoine Lavoisier, Dmytro Mendeleev, Benoit Clapeyron, Ipolit Gromeka, Horace Lamb, Jean Fourier, Henry Darcy, Isaac Newton, Gustav Kirchhoff, Adolf Fick, and Lars Onsager [1-8]. (Eq. 1):

To write down thermodynamically the last equation in (1), \tilde{s} entropy is assumed as the mentioned \tilde{s} fluid; K is considered as T temperature; and thermal conductivity λ_T is a diagonal element of matrix coefficient \tilde{L}_{ij} .

Equation system (1) is terminated by synchronization of the number of equations relative to the number of the unknowns of the application dual extraction task using the “fluid-geoheat” well system while achieving the development of equation system (i.e. synthesis) from the phenomenological matrix-type Onsager ratios.

$$\begin{aligned} \frac{\partial \bar{v}}{\partial t} &= \bar{F}_m - \frac{1}{\rho} \cdot \text{grad } P + \gamma \cdot \nabla^2 \bar{v} && \text{is motion equation (Euler, Navier, and Stokes);} \\ \frac{m \cdot \partial \rho}{\partial t} + \text{div}(\rho \cdot \bar{v}) &= 0 && \text{is continuity equation (Lavoisier);} \\ \frac{P}{\rho} &= z \cdot R \cdot T && \text{is a state equation (Mendeleyev and Clapeyron);} \\ \text{rot} \frac{\partial \bar{v}}{\partial t} &= \frac{\partial \bar{v}}{\partial t} - \text{grad} \left(\frac{v^2}{2} + \frac{P}{\rho} \right) + 2 \cdot \bar{\omega} \cdot \bar{v} && \text{is equation of turbulence (Gromeka and Lamb);} \\ &&& (1) \\ \rho \cdot \frac{\partial E}{\partial t} + P \cdot \text{div}(\bar{v}) &= \varepsilon && \text{is energy equation (Fourier and Newton);} \\ \bar{v} &= \frac{k \cdot k_f}{\mu} (\bar{\nabla} P - \rho \cdot \bar{g} \cdot \nabla h) && \text{is filtration equation (Darcy);} \\ \frac{\partial T}{\partial t} &= \frac{Q_v}{C_v \cdot \rho} + D_{TG} \cdot \text{div}(\text{grad } T) && \text{is thermodiffusion equation (Fick); and} \\ \ddot{I} &= \frac{1}{K} \cdot \sum_{i,j} \ddot{q}_{i,j} \ddot{L}_{ij} \ddot{X}_i \cdot \ddot{X}_j && \text{is equation of mutual influence of processes} \\ &&& \text{(Kirchhoff, and Onsager)} \end{aligned}$$

where $\ddot{q} = 2$ is the number of the processes (i.e. hydraulic process and thermodynamic one); K is reduction factor for measurement units; \ddot{L}_{ij} is coefficient of conditional and linear dependence between motive power of a certain type (\ddot{X}_i and \ddot{X}_j) and fluid flattening up to a balance state of a process belonging to a correspondent \ddot{I} type. Other specifications are given in the next section.

Paper [11] proposes similar mathematical model being improved to some extent as related to the use of Rubinstein model [8-9]; moreover, it takes into consideration the fact that influence of hydraulic conductivity on the reservoir processes is by 3-4 orders more to compare with thermal conductivity [10]:

$$\begin{aligned} \text{div}(\text{grad } P) &\cong \frac{1}{x_p} \cdot \frac{\partial P}{\partial t}; \\ \frac{\partial \bar{v}}{\partial t} &= \bar{g} - \frac{1}{\rho} \cdot \text{grad } P + \gamma \cdot \nabla^2 \bar{v} && \text{is motion equation;} \\ \frac{m \partial \rho}{\partial t} + \text{div}(\rho \cdot \bar{v}) &= 0 && \text{is continuity equation;} \\ \frac{P}{\rho} &= R \cdot T && \text{is state equation;} && (2) \\ \rho \cdot \frac{\partial E}{\partial T} + P \cdot \text{div}(\bar{v}) &= \varepsilon && \text{is energy equation;} \\ v &= \frac{k \cdot k_f}{\mu} (\nabla P - \rho \cdot g \cdot \nabla h) && \text{is filtration equation; and} \\ \frac{\partial T}{\partial t} + \frac{Q_v}{c_v \cdot \rho} + D_{TG} \cdot \text{div}(\overline{\text{grad}} T) - \overline{\text{grad}}(v \cdot T) &&& \text{is thermal exchange equation.} \end{aligned}$$

Pressure, temperature, density, velocity, consumption, and time (i.e. $P, T, \rho, \bar{v}, Q_v, t$) are unknowns within the equation system (2); i.e. its solution is unambiguous in terms of the initial conditions and boundary ones. In this context, P varies within the range of formation pressures P_{pl} from a boundary of fluid drainage within a well to its bottom P_{bh} with an average $P_{pl_{av}}$ value. T temperature also varies between the mentioned geometrical area with reservoir temperature T_{pl} ; and bottomhole temperature T_{bh} ; in this context, average value is $T_{pl_{av}}$.

Equation system (2) simulates adequately heat transfer fluids; however, it involves no members taking into consideration biphasity and thermodiffusion within the heat exchanger. Among other things, neither simulation nor determination of thermal efficiency $W_T = M_q \cdot C_p (T_{wh} - T_{back})$

considered direct (Joule–Thomson) effect and inverse (mechanocaloric) effect which dropped accuracy of the calculations [10-11]. In this context, greater attention was paid to thermoelastic filtration nature being less important for thermal efficiency of a production well to compare with heat exchange nature within a seam [10]. Heat exchange nature within thin seams is stipulated by thermodiffusion specificity with a permeable reservoir differing significantly in terms of motion of reservoir water interfusion or other fluids with hydrocarbon gas [12]. The conditions prevent from theoretical studies of thermal efficiency dependence upon the bottomhole temperature.

Thus, the objective is to specify methods to calculate thermal efficiency of hydrocarbon well system “fluid-geoheat”. To achieve the objective, following research problems have been formulated:

- develop analytical mathematical thermal efficiency model of dual well production system “fluid-geoheat”; and
- analyze dependence of thermal efficiency W_T in the process of transfer medium (i.e. gas-liquid fluid) extraction upon bottomhole temperature T_{bh} and reservoir pressure P_{bh} .

2. Experimental

2.1. Mathematical model

Table 1 demonstrates the output data as well as the description of simulation parameters of a thin seam with hydrocarbon fluids under the conditions of reversible heat carrier injection of the integrated mining using “fluid-geoheat” system for a well of Yefremivske GCF.

Table 1. Output data to simulate a thin seam with hydrocarbon fluids under the conditions of reversible heat carrier injection of the integrated mining cycle using “fluid-geoheat” system for a well of Yefremivske GCF

Units name	Value range
C_p is specific heat capacity (J/kg K);	2200-2500
K_t is total heat transfer coefficient (W/m ² K);	0.5 – 1
R_k is fluid drainage diameter across the reservoir (m);	300
k is permeability of reservoir layer (m ²);	3.00E-13
H is (m);	3500
M_q is mass flow rate (kg/s);	20
W is power – heat flow rate (J/s);	–
Re is Reynolds number;	10E5-10E7
F is cross-sectional area (m ²);	1
t is time (s);	–
V is volume (m ³);	–
v is flow velocity of the fluid (m/s);	2-20
λ_t is thermal conductivity of material (W/m K);	2
ρ is density (kg/m ³);	30-100
ρ_{st} is density under standard conditions (kg/m ³);	0.76
x is distance (m);	0-3500
P_{wh} is absolute upstream pressure (Pa);	1000000
P_{bh} is absolute downstream pressure (Pa);	20000000
L_p is length of pipe (m);	3500
d is inside diameter of pipe (m);	0.073
z is factor of compressibility;	0.85-0.95
Δ is specific gravity, relative density;	0.59
λ is factor of hydraulic resistance;	0.01-0.03
ΔH is a difference of heights (m);	0-3500
P_{pl} is rock layer pressure (Pa);	15E6-25E6
T_{pl} is rock layer temperature (K);	360
D_j is Joule-Thomson coefficient (K/MPa);	1-5
R is gas constant (J/(kg·K));	486
T_{pl} is ground temperature (K);	280-380
P is pressure (Pa);	10E5-60E6
T is temperature, (K);	200-400

Units name	Value range
P_1 is pressure absolute the source (Pa);	10000000
P_2 is pressure absolute the receiver (Pa) ;	1000000
μ is dynamic viscosity (Pa·S);	1,00E-04
M is molar mass (kg/mol);	17-20
k_e is roughness inside the pipe (m);	0.0003
φ_t is nonisothermal correction factor;	0.8
h is high of layer (m);	20
P_{at} is atmospheric pressure (Pa);	100000
T_{at} is atmospheric temperature (K);	293
A_{av} is linear coefficient of reservoir filtration resistance ((MPa ²)/(Th-nd.m ³ /day));	0.6-0.9
B_{av} is second coefficients of reservoir filtration resistance (((MPa ²)/(Th-nd.m ³ /day) ²));	0.001-0.005
P_{pc} is pseudocritical pressure (Pa);	4.63E6
T_{pc} is pseudocritical temperature (K);	198
R_c is radius off well productive pipe (m);	0.1
P_{bh} is borehole pressure (Pa);	20000000
P_{wh} is wellhead pressure (Pa);	15000000
S is skin factor;	0
z_{st} is compressibility under standard conditions;	0.9
k_o is coefficient of accommodation;	0.022
j_n is degree parameters in the Newselt number equation.	0.5
P_{rav} is avarage pressure of rock layer (Pa);	20E6-30E6
T_{rav} is average temperature of rock layer (K);	350-370
R_{air} is gas constant air (J/kg K);	2870
R_u is universal gas constant (J/Kmol K);	8314
α is coefficient of thermal expansion of fluid;	0.001
S_1 is skin factor for coefficient A	0
S_2 is skin factor for coefficient B	0
g is Accelerating gravity (m/s ²)	9.8
N_{ng} is factor in the fluid flow equation to the well bore	1-2
β is Coefficient of macro-rigidity of the rock of the productive formation	2.2
K_a is The coefficient of annihilation of the effect of throttling	0-1
β_a is Coefficient of annihilation of change of thermal conductivity depending on dynamic temperature	0-1
K_{to} is Heat transfer coefficient for primitive geothermal gradients	0.5-4

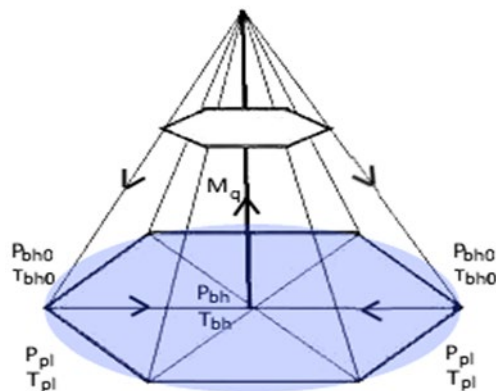


Figure 1. Model of a thin fluid-saturated seam in the context of operation of "fluid-geoheat" type system with geometry of radial planar inflow between the branched bottomhole well elements and a primary bottomhole

Consider physical model seam-adjointing rocks in the form of a thin cylinder (Fig. 1) where a heat carrier moves from walls to a centre and to the branched bottomhole. The motion results in the intensive heat exchange between the adjoining rocks and a seam; and diffusion blending of a fluid being injected as well as a fluid being mined within walls for further extraction and surface use of heat and hydrocarbon raw material.

Heat exchanger of an upper part of the well bottomhole (acceptor and utilizing), and heat-exchange disk plate (reservoir donor heat exchanger) are shown one below the other being inscribed with hexagons. Heat carrier moves from a branching point of an upper part of a bottomhole to the disk walls along the inclined drain holes (Fig. 1); then,

it passes through the plate disk, and is mined by means of conditionally vertical column from the disk centre. In this context, accepting heat interchanger of a thermal pump may be mounted at an arbitrary level (for instance, as it is shown in the Figure) as well as at a surface near the well mouth. It is understood from Fig. 1 that pressure on the disk plate sides P_{bh0} should exceed central pressure P_{bh} to concentrate the heated transfer medium and perform extraction through the primary bottomhole with mass flow rate M_q .

While agreeing of the known formulas for a cylinder with vertical axis (of a thin disk type where ends are oriented within the horizontal plane of a reservoir) for quasi-steady centrifugal fluid drainage towards cylindrical side wall at R_k distance, and defining of simplifications as for the consistent conditionally homogeneous porous formations and gas-liquid fluid flows [7-8], consideration of the extra condition makes it possible to develop an analytical mathematical model in terms of cylindrical coordinates. The model involves equations of continuity; motion; filtration; state; mechanical energy or inflow; heat exchange; and thermodiffusion in the form of:

$$M_q = 2 \cdot \pi \cdot r \cdot h \cdot \rho(r) \cdot v(r) - const \quad \text{being continuity;}$$

$$\frac{M_q \cdot \mu \cdot \ln \frac{r}{R_c}}{\pi \cdot k \cdot h \cdot \rho(r)} - \Delta P(r) = \rho(r) \cdot g \cdot h + \rho(r) \cdot z \cdot R \cdot \Delta T(r) \quad \text{being motion;}$$

$$v(r) = -\frac{2 \cdot \pi \cdot k \cdot h}{\mu \cdot \ln \frac{r}{R_c}} (P(r) - P_{bh}) \quad \text{being filtration;}$$

$$\frac{P(r)}{\rho(r)} = z \cdot R \cdot T(r) \quad \text{being state;} \quad (3)$$

$$(P_{pl})^2 - (P_{bh})^2 = A_{av} \cdot (M_q) + B_{av} \cdot (M_q)^2 + (\rho_{av} \cdot g \cdot h)^2 \quad \text{being inflow;}$$

$$M_q \cdot C_p \cdot (T_{bh} - T_{pl} + (P_{bh} - P_{pl}) \cdot D_j) = 2\pi \cdot \int_0^{R_k} (K_T(r) \cdot r \cdot (T_{pl}(r) - T(r))) dr \quad \text{heat exchange; and}$$

$$\frac{Q_v}{c_v \cdot \rho} = -D_{TT} \cdot \text{div}(\text{grad } T(r)) = D_{TT} \cdot \text{div} \left(\frac{2}{R_k} - \frac{2}{r} \right) \quad \text{being thermodiffusion within a heat exchanger;}$$

where $\text{grad } T(r) = -\left(\frac{2}{R_k} - \frac{2}{r}\right) \geq 0$ are following boundary conditions: $\text{grad } T(R_c) = -\left(\frac{2}{R_k} - \frac{2}{R_c}\right) \geq 0$ being temperature differential within a bottomhole; $\text{grad } T(R_k) = 0$ being temperature differential within walls; $\Delta T(R_c) = T_{result} - T_{bh} \geq 0$ being temperature difference of heat carrier input and output within a bottomhole where T_{bh} is a discharge temperature; $\Delta T(R_k) = 0$ being temperature equalization within walls; $\Delta P(R_k) = 0$ being equalization of pressure for a flow injection and extraction within walls; and $\Delta P(R_c) = P_{result} - P_{bh} \leq 0$ being total loss of input-output pressure where P_{bh} is discharge pressure.

Model (3) represents motion equation as that one based upon pressure balance. Moreover, equations of mechanical energy (inflow), heat exchange, and thermodiffusion serve as the boundary conditions. Heat exchange equation [9] takes into consideration external the rock heat income, i.e. thermodiffusion within a reservoir boundary ϵ . The last thermodiffusion equation within (3) system is only derived for the specific consideration of thermodiffusion features within a heat exchanger. Mechanical energy equation is also known as an inflow equation. It is based upon the quadratic pressure (according to Forchheimer) involving fluid biphasity. To compare with the available analogues to simulate quasi-steady processes, the innovations are fundamental in model (3) design; however, the initial conditions were selected similarly to [10-11].

Processes within a heat exchanger with counterflows with backward flows as well as pressure and temperature gradients of backward flows $\Delta T(r)$ and $\Delta P(r)$ can be calculated using

different mathematical approaches if one assumes that the flows exchange heat instead of being interacting. We propose the following. In the context of a theory of non-equilibrium thermodynamics [12], recommendations by Onsager [6], and analysis of stationary processes by K. Denbigh [6], ratio between thermobaric gradients within a thin heat exchanger may be specified with the help of the known equation of thermomechanical effect:

$$\frac{\Delta P(r)}{\Delta T(r)} = \frac{Q_v}{V \cdot T(r)} \rightarrow \Delta P(r) = \frac{\rho(r) \cdot C_p (\Delta T)^2}{T(r)} \quad (4)$$

complemented by thermodiffusion conditions $\frac{\Delta P}{\Delta T} = \frac{P}{2T}$ for operation of capillaries as well as thin membranes.

It should be mentioned that records of equation system (3) and equation (4) are slightly different from those represented in [8-11] in the context of fluid compressibility; and specificity of heat-mass-exchange processes for geometry of a thin heat exchanger disk. System (3), consisting of seven equations, is the completely closed equation systems relative to the unknown seven parameters $P(r), T(r), \rho(r), v(r), \Delta T(r), r$.

It should be taken into considerations that in terms of the formation pressure drop, the current fluid reserves, being mined, are determined using equation [8]:

$$Q_m = \frac{Q_{pr}}{\frac{P_{st} - P_{rav}}{z_{st}}} \cdot \frac{P_{st}}{z} - Q_{pr} \quad (5)$$

where Q_{st} are initial reserves; Q_{pr} is current accumulated extraction; P_{st} and P_{rav} are initial formation pressure and the current one respectively; Z_{st} and Z are initial compressibility coefficient and the current one respectively; and Q_m is current fluid reserves being mined. Record equation of continuity as a material balance in more detail:

$$Q_m = F \cdot h \cdot m \cdot \rho \cdot \xi \cdot f \cdot K_p = \frac{Q_{pr}}{\frac{P_{st} - P_{rav}}{z_{st}}} \cdot \frac{P_{st}}{z} - Q_{pr} \quad (6)$$

where F is gas and oil potential area corresponding to fluid drainage area, m^2 ; h is efficient fluid saturated formation, m ; m is open porosity, particles of unit; ρ is fluid density, kg/m^3 ; ξ is rock fluid saturation, particles of unit; f is contraction coefficient, particles of unit (corrected to recalculate volume from formation conditions to operational (i.e. average) conditions or to surface ones (in terms of an open flow); P_{rav} is current average formation pressure; P_{st} is formation pressure at the initial stage of oil and gas geothermal deposit mining; K_p is fluid output coefficient, particles of unit; z_{st} is compressibility coefficient of actual gas-fluid formation fluid as at the start of deposit mining, particles of unit; and Q_{pr} is increase in formation fluid reserves from the beginning of productive deposit mining, total fluid extraction.

Hence, (3)-(6) equations are certain complete mathematical model of a conditionally thin seam operation involving its depletion as well as the current formation pressure.

3. Results

Theoretical originality of the specified methods is in the integral structure of energy equation recording as well as in the consideration of efficient temperature within the radius axis of cylindrical coordinates. The abovementioned differs from its analogues owing to the transition of temperature gradient, arising as a result of Joule-Thompson effect, to the right-hand member (including a corresponding positive consequence according to Fourier law) to the thermal efficiency-external heat input balance. The new theoretical knowledge is based upon the use of efficient temperature within an equation of the second Fourier-Newton law of heat conduction rather than theoretical temperature to involve Joule-Thompson effect as additional heat pumping from the neighbouring adjoining rocks to a liquid-gas mixture.

Practical value of taking into consideration the Joule-Thompson control of liquid-gas of hydrocarbon fluid within the productive formation in terms of radial filtration is to increase thermal efficiency while using the effect of extra heat pumping from the productive seam floor.

Evaluate changes in thermal efficiency depending upon the working pressure and temperature within the seam while ignoring internal heat loss which is valid for intensive laminary flows as well as for homogenous hydrocarbon mixtures [13]. To do that, stage one determined discharge pressure and temperature at a late mining stage, and partial depletion of gas-condensate deposit using extraction data of the selected deposit as well as (4)-(6) formulas. Discharge pressure is $P_{bho} = 21$ MPa; temperature is $T_{bho} = 300$ K. On their way from a seam towards a bottomhole, compressibility z , heat capacity at constant pressure C_p , dynamic viscosity μ , density of natural hydrocarbon, and Joule-Thompson effect D_j will vary significantly depending upon P , being actual pressure, and T temperature T [9]. They can be identified for natural gas where methane content is more than 90%, molar weight is M -const, and density, being ρ_{st} - const under the reference conditions, using following empiric functional dependences by Platonov-Gurevich, Starling-Ellington, and Lurie [10-11]:

$$z(P, T, \rho_{st}) = \frac{0.1 \cdot P}{P_{pc}(\rho_{st})} + \left[0.4 \cdot \log\left(\frac{T}{T_{pc}(\rho_{st})}\right) + 0.73 \right] \frac{P}{P_{pc}(\rho_{st})} \quad (7)$$

$$\mu(P, T, M) = \frac{(9.41 + 0.02) \cdot (1.8 \cdot T)}{209 + 19 \cdot M + 1.8 \cdot T} \cdot \text{EXP} \left(\left(3.5 + \frac{547.8}{1.8 \cdot T} + 0.01 \cdot M \right) \cdot \left(\frac{P \cdot 10^3}{z(P, T, \rho_{st}) \cdot \frac{8314.3}{M} \cdot T} \right)^{\left(2.44 \cdot \left(3.5 + \frac{547.8}{1.8 \cdot T} + 0.01 \cdot M \right) \right)} \right) \quad (8)$$

$$C_p(P, T, M) = \left(900 \cdot 1.014^{T-273} \cdot T^{-0.7} + 2170 \cdot 1.015^{P \cdot 10^{-6}} \cdot P^{0.0214} \right) \left(\frac{R_{air} \cdot M}{0.6 \cdot R \mu} \right)^{0.025} \quad (9)$$

$$D_j(P, T, M) = \frac{1 - a \cdot T}{C_p(P, T, M) \cdot \rho(P, T, M)} \quad (10)$$

Where $T_{pc}(\rho_{st}) = 88.25 \cdot (0.9915 + 1.759 \cdot \rho_{st})$ and $P_{pc}(\rho_{st}) = 2.9585 \cdot (1.608 - 0.05994 \cdot \rho_{st})$ are pseudocritical parameters of a gas-condensate mixture; $\rho(P, T, M) = \frac{P}{z(P, T, M) \cdot R \cdot T}$ is the mixture density under operating conditions.

After transition from the initial conditions and boundary conditions to operating ones, in accordance with (6), analytical model from (Fig. 1) representation uses following equation system consisting of four basic equations (i.e. formulas of continuity, state, filtration, and energy) to derive not radial functions but specific actual values of gas dynamic parameters within a definite points at a $r=0-300$ m distance from a bottom where central part of a disk plate is discharged:

$$\begin{aligned} M_q &= 2 \cdot \pi \cdot r \cdot h \cdot \rho \cdot m \cdot v - \text{const} && \text{is continuity equation;} \\ v &= -\frac{2 \cdot \pi \cdot k \cdot h}{\mu \cdot r \cdot \ln \frac{r}{R_c}} (P - P_{bho}) && \text{is filtration equation;} \\ \frac{P}{\rho} &= z(P, T, M) \cdot R \cdot T && \text{is state equation;} \end{aligned} \quad (11)$$

$$W = M_q \cdot C_p(P, T, M) \cdot (T_{bh0} - T) = 2\pi \cdot \int_{Rc}^r (K_T \cdot x \cdot (T_{pl}) + (P_{bh0} - P) \cdot D_j(P, T, M) \cdot K_a) dx \quad \text{is energy equation}$$

Equation (11) demonstrates that throttling effect has been transferred to the right-hand member of Fourier law record which makes it possible to consider intensification of heat pumping within each point at r distance. The equation system has four unknown parameters (i.e. pressure P ; temperature T ; density ρ ; and filtration velocity v). Equation four is thermal efficiency W calculated according to pressure difference and temperature gradients towards the adjoining rocks on the way from injection point $r = 0$ to a point with its coordinate on r radius. It is mass flow rate considering heat carrier circulation M_q – const as that stabilized by a pump; the abovementioned helped reduce the number of the equations down four. D_j formula of hydrocarbon fluids of gas-condensate fluids was applied for final calculations in the form of [9]:

$$D_j(P, T, M) = \frac{K_a}{C_p(P, T, M) \cdot 10^{-3}} \left(\frac{0.98 \cdot 10^6}{T^2} - \frac{1}{\rho(P, T, M)} \right) = K_a \left(\left(24.96 - 20.3 \cdot \frac{T}{T_{pc}} \right)^2 + \left(5.6679692 \cdot \frac{T}{T_{pc}} + 16.89 \left(\frac{T}{T_{pc}} \right)^2 \right) \cdot \frac{P}{P_{pc}} + \left(-4.66 + 14.68 \cdot \frac{T}{T_{pc}} + 13.39 \cdot \left(\frac{T}{T_{pc}} \right)^2 \right) \cdot \left(\frac{P}{P_{pc}} \right)^2 + \left(0.56891 \cdot \frac{T}{T_{pc}} + 1.79 \cdot \left(\frac{T}{T_{pc}} \right)^2 \right) \cdot \left(\frac{P}{P_{pc}} \right)^3 \right) \quad (12)$$

The initial conditions to solve nonlinear system (11-12) of analytical mathematical model of a conditionally thin seam with centrifugal discharge of hydrocarbon fluid of a reverse cycle (i.e. from the central share to walls with mining along their line) have been selected as those ones represented in Fig. 1. Computational methods by Runge-Kutta, Quasi-Newton, and others have been applied with their automatical transfer during each calculation stage. Matcad 15 software, used for the purpose, has helped provide maximum calculation accuracy. The calculation process involved (9) formula for C_p and (12) formula to determine D_j finally without any additional simplifications while solving (1) within the whole r range. Fig. 2 shows temperature and pressure $T(r)$, $P(r)$, $T(r, D_j)$, and $P(r, D_j)$ dependences upon r radius involving extra heat pumping from a seam floor and ignoring it. The fact is represented in the function records.

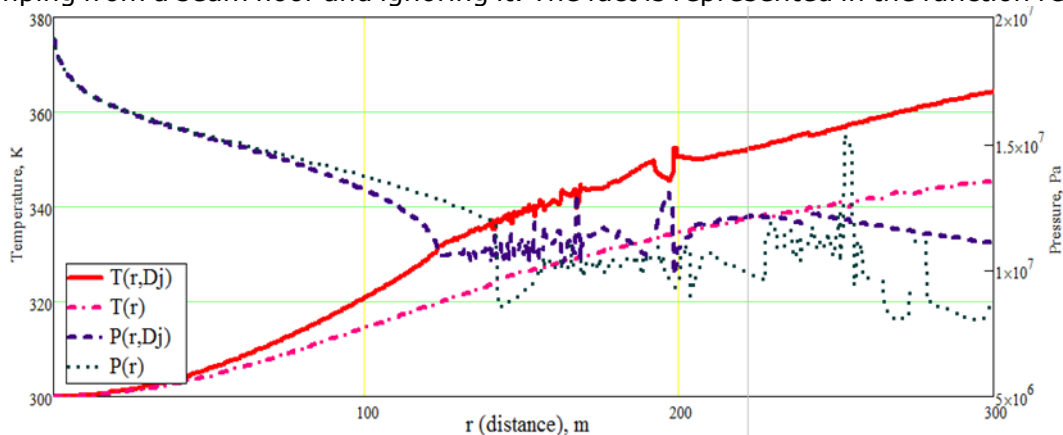


Figure 2. $T(r)$, $P(r)$, $T(r, D_j)$, and $P(r, D_j)$ temperature and pressure dependences upon r radius involving extra heat pumping from a seam floor and ignoring it

If Joule-Thompson effect is involved then extra pumping of liquid-gas mixture may be calculated approximately using other simplified methods; however, the proposed technique has determined accurate values for each point within r axis to enable qualitative comparative thermal efficiency evaluation between the liquids with active manifestation of the effect, and

those with minor one. Consideration of one-side heat inflow to imaginary plate (from a floor) is the important feature of a thin disk plate simulation since it goes without saying that heat transfer area should not add twice in essence on the preevaluated coefficient heat transfer coefficient for a plate operating unidirectionally [15-16].

Figure 2 explains that temperature at a distance of the selected drainage radius, being $R_k = 300$ m, differs by 18.5 K in alternations with complete consideration of Joule-Thompson effect as well as in terms of its nonavailability. Calculate thermal efficiency if discharge temperature is $T_{bh0} = 300$ K; one thermal capacity, being $C_p = 2500$ J/kgK, is available; and mass flow rate of the example is $M_q = 20$ kg/s:

$$W1 = M_q \cdot C_p \cdot (T_{bh0} - T(r, Dj)) = 20 \cdot 2500 \cdot (300 - 365) = 3.25 \text{ MW}; \text{ and}$$

$$W2 = M_q \cdot C_p \cdot (T_{bh0} - T(r)) = 20 \cdot 2500 \cdot (300 - 346.5) = 2.32 \text{ MW}.$$

In the context of approximate calculations, Joule-Thompson effect is taken into consideration according to pressure difference fact. The example obtains 10 MPa pressure difference. Hence, at a 300 m distance, the temperature may be evaluated in terms of such a discrete value of the simplified approach as $Dj = 2.5$ K/MPa with down to 25 K drop which stipulates an error in the thermal efficiency increase:

$$W3 = M_q \cdot C_p \cdot (T_{bh0} - T(r, 2.5)) = 20 \cdot 2500 \cdot (300 - 346.5 - 25) = 3.57 \text{ MW}.$$

Hence, calculations of the heated fluid extraction within the walls where distance between a discharge point and a heat extraction point is 300 m have helped obtain 33% thermal efficiency difference relative to the average value of the thermal efficiency (i.e. between alternatives of throttle considering W2 and its ignoring W1). The difference in the overvaluation of thermal efficiency in terms of the considered value of Joule-Thomson coefficient relative to average pressure and temperature amount as well as to the specified on the inverse thermo-mechanical effect W3 (as for the really estimated according to the proposed methods W2) is 9.4%. It should be noted that within the range, thermal efficiency is supported by massive practice of geothermal power resource production from oil and gas wells [15] which supports reliability of the carried out studies as well as their applied relevance. Moreover, consideration of the heat value inflow only from a floor rather than double heat inflow value, being typical for a massive bed, is quite an important result of the research since it reflects adequately the specificity of heat-mass-exchange for a conditionally thin seam.

Fig. 2 demonstrates almost linear dependence of temperature dependence upon r coordinate, i.e. exponential pressure drop. In this context, changes in pressure within the walls of the considered seam are 1-1.2 MPa irrespective of the throttle effect consideration or ignoring. In terms of classical consideration of a throttle effect, temperature difference within the walls where distance is 300 m could be less than 4 K. Seemingly, the fact may be ignored; however, relying upon the fact that in the context of the considered 20 kg/s mass flow rate of a good gas-condensate well of Yefremivske GCF, each degree of fluid heating generates specific energy power being 0.2 MW. Thus, it is not permitted to ignore throttle effect for high-rate gas-condensate wells [16-20].

It should be mentioned that in future filtration equations of fluid in pores and capillaries must take into consideration the fluid structure within a wall-adjacent area, i.e. availability of so-called thin films having tight adhesive connection with a solid phase; intermediate adsorption layers; and gravitational fluid layers not connected with pore walls. It is especially important while balancing dimensions of pores and fissures (i.e. interior cross section of a filtration flow) and thickness of wall-adjacent and adsorption fluid films having abnormal characteristics in terms of density and viscosity [22]. Fig. 3 demonstrates changes in fluid density and filtration velocity depending upon r radius for the performed analytical modeling.

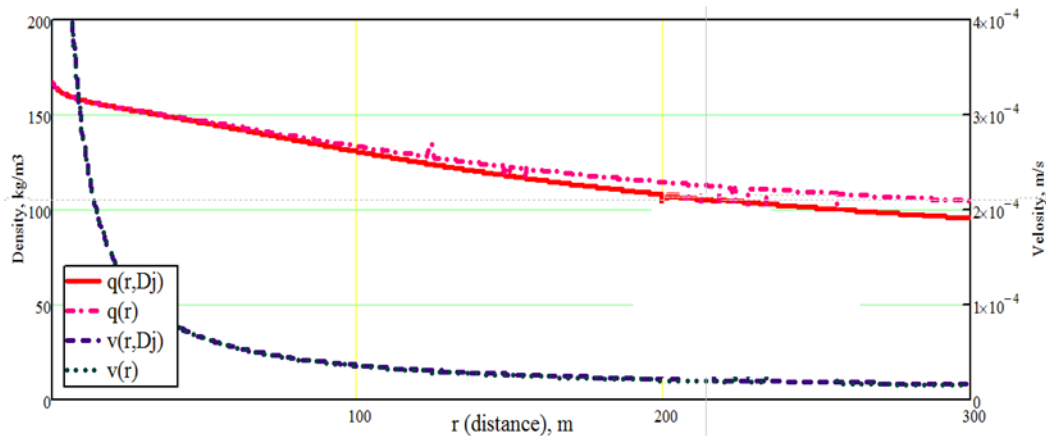


Figure 3. Dependence of density and filtration velocity $q(r)$, $v(r)$, $q(r, Dj)$, and $v(r, Dj)$ upon r radius involving extra heat pumping from a seam floor and ignoring it

Fig. 3 explains that throttle effect consideration cannot vary the velocity dependence curve upon a coordinate while moving to the centre; density splitting is only 2.5% within $r = 300$ m point. Hence, Fig. 3 supports the study of two variants with similar values (under analogous conditions) of the velocity and density of heat-carrier fluid.

Thus, the results of mathematical modeling of a thin centrifugal-type reservoir with the external heat exchange and thermodiffusion from a floor are of theoretical and practical importance. Among other things, dependence of thermal efficiency of underground heat exchanger upon actual pressure and temperature has been analyzed. The dependence trends upward if Joule-Thompson effect manifestation within a hydrocarbon saturated seam has been taken into consideration. The consideration may rely upon various known methodological approaches. The methods, proposed and substantiated by the authors, make it possible to obtain the most accurate predicted thermal efficiency data of a dual production well system "fluid-geoheat".

One more important result is that the obtained thermal efficiency values of hydrocarbon well, from which raw material and geothermal energy are extracted simultaneously, are in line with the experimental results [23-25] while heat capacity recalculating depending upon the considered ratio between the hydrocarbon mixture composition and water.

4. Conclusions

Analytical mathematical model of thermal efficiency of a dual well "fluid-geoheat" system with centrifugal radial filtration through a conditionally thin hydrocarbon saturated seam has been developed. It has been demonstrated that relative to a variant, considering throttle effect, and a variant with Joule-Thompson coefficient discrete substitution, calculation accuracy of the well thermal efficiency increases by 33% and 9% respectively. It has been proved that it is expedient for a conditionally thin seam to use efficient temperature in terms of a heat exchange equation with Fourier substitution as related to heat transfer; it is practical to take into consideration heat inflow in terms of unidirectional inflow to an imaginary disk plate from a seam floor. Increase in numerical evaluations of a well thermal efficiency in terms of the proposed calculation method use is stipulated by the consideration of extra heat pumping effect at the expense of throttle effect from the rock adjoining the productive formation.

Thermal efficiency analysis within a radial coordinate axis depending upon pressure and temperature demonstrates linear dependence of temperature being in turn contingent on the radial coordinate. The proposed methods make it possible to evaluate thermal efficiency of a "fluid-geoheat" system more accurately (i.e. by 33%), and by 9% more relative to consideration variants with substitution of discrete values on pressure drop within the radial planar flow type as well as the integrally evaluated thermomechanical effect.

Symbols

C_p – specific heat capacity;
 K_t – total heat transfer coefficient;
 R_k – fluid drainage diameter across the reservoir;
 k – permeability of reservoir layer;
 M_q – mass flow rate;
 W – power – heat flow rate;
 Re – Reynolds number;
 F – cross-sectional area;
 t – time;
 V – volume;
 v – flow velocity of the fluid;
 λ_t – thermal conductivity of material;
 ρ – density;
 ρ_{st} – density under standard conditions;
 x – distance;
 P_{wh} – absolute upstream pressure;
 P_{bh} – absolute downstream pressure;
 L_p – length of pipe;
 d – inside diameter of pipe;
 z – factor of compressibility;
 Δ – specific gravity, relative density;
 λ – factor of hydraulic resistance;
 ΔH – a difference of heights;
 P_{pl} – rock layer pressure;
 T_{pl} – rock layer temperature;
 D_j – Joule-Thomson coefficient;
 R – gas constant;
 T_{pl} – ground temperature;
 P – pressure;
 T – temperature;
 P_1 – pressure absolute the source;
 P_2 – pressure absolute the receiver;
 μ – dynamic viscosity;
 M – molar mass;
 k_e – roughness inside the pipe;
 φ_t – non-isothermal correction factor;
 h – high of layer;
 P_{at} – atmospheric pressure;
 T_{at} – atmospheric temperature;
 A_{av} – linear coefficient of reservoir filtration resistance;
 B_{av} – second coefficients of reservoir filtration resistance;
 P_{pc} – pseudocritical pressure;
 T_{pc} – pseudocritical temperature;
 R_c – radius off well productive pipe;
 P_{bh} – borehole pressure;
 P_{wh} – wellhead pressure;
 S – skin factor;
 Z_{st} – compressibility under standard conditions;
 k_o – coefficient of accommodation;
 $j n$ – degree parameters in the Newselt number equation.
 P_{rav} – average pressure of rock layer;
 T_{rav} – average temperature of rock layer;
 R_{air} – gas constant air;
 R_u – universal gas constant (J/Kmol K);
 α – coefficient of thermal expansion of fluid;
 S_1 – skin factor for coefficient A;
 S_2 – skin factor for coefficient B;
 g – Accelerating gravity;

N_{ng} – factor in the fluid flow equation to the well bore;
 β – Coefficient of macro-rigidity of the rock of the productive formation;
 K_a – The coefficient of annihilation of the effect of throttling;
 β_a – Coefficient of annihilation of change of thermal conductivity depending on dynamic temperature;
 K_{to} – Heat transfer coefficient for primitive geothermal gradients.

References

- [1] Deyk LP. Prakticheskiy inzhiniring rezervuarov. Moskva-Izhevsk: Institut komp'yuternykh isledovaniy.
- [2] Karpenko VM, Starodub YP. Research of geothermal energy parameters in deep wells. *Geodynamics*, 2017; 1(22): 85-97.
- [3] Kravchenko IP. Protsesy ta systemy peretvorennia heotermalnoi enerhii vyroblyenykh naftovykh i hazovykh rodovyshch. PhD Dissertation. NAN Ukrainy, Instytut vidnovliuvalnoi enerhetyky. Kyiv: Instytut vidnovliuvalnoi enerhetyky.
- [4] Kulikova T, Markov P, Solonin V. Simulation of Heat Transfer to the Gas Coolant with Low Prandtl Number Value. *Science and Education of the Bauman MSTU*. <https://doi.org/10.7463/0615.0780763>.
- [5] Brehme M, Bauer K, Nukman M, Regenspurg S. Self-organizing maps in geothermal exploration—A new approach for understanding geochemical processes and fluid evolution. *Journal of Volcanology and Geothermal Research*, 336, 2017, p.19-32.
- [6] Onsager L. Reciprocal Relations in Irreversible Processes. II. *Physical Review*, 38(12), 1931, p. 2265-2279.
- [7] Lyal'ko VI, Mitnik MM. Issledovanie protsessov perenosa tepla i veshchestva v zemnoy kore. 1978. Kiev: Naukova dumka.
- [8] Shmyglevskiy YuD. Analiticheskie issledovaniya dinamiki gaza i zhidkosti. Moskva. Editorial URSS, 1999. p. 232.
- [9] Fyk M, Biletskyi V, Abbood M, Al-Sultan M, Abdullatif H, Shapchenko Y. Modeling of the lifting of a heat transfer agent in a geothermal well of a gas condensate deposit. *Mining of Mineral Deposits*, 14(2), 2020. 66–74.
- [10] Morozov YuP. Dobycha geotermal'nykh resursov i akumulirovanie teploty v podzemnykh gorizontakh. 2017. Kiev, Naukova dumka, 200.
- [11] Alkhasov AB, Alkhasova DA. Heat Exchangers for Utilization of the Heat of High-Temperature Geothermal Brines. *Thermal Engineering*, 2018; 65(3): 155-159.
- [12] de Groot SP. Termodinamika neobratimyykh protsessov. 1956. Moskva. Gosudarstvennoe izdatel'stvo tekhniko-teoreticheskoy literatury, 281.
- [13] Biletskyi V, Shendrik T, Sergeev P. The estimation of effect of the dust-proof respirators' protective efficiency upon the mining workers' dust load. *Geomechanical Processes During Underground Mining*, 2012; 187-190.
- [14] Fyk M, Biletskyi V, Ryshchenko I, Abbood M. Improving the geometric topology of geothermal heat exchangers in oil bore-holes. *E3S Web of Conferences*, 2019; 123: 01023.
- [15] Fyk M, Biletskyi V, Abbood M. Resource evaluation of geothermal power plant under the conditions of carboniferous deposits usage in the Dnipro-Donetsk depression. *E3S Web of Conferences*, 60. 2018. <https://doi.org/10.1051/e3sconf/20186000006>
- [16] Desideri U, Bidini G. Study of possible optimisation criteria for geothermal power plants. *Energy Conversion and Management*, 1997; 38(15-17): 1681-1691.
- [17] Di Maria F. Design and off design pipe network geothermal power plant analysis with power pipe simulator. *Energy Conversion and Management*, 41(12), 2000. 1223-35.
- [18] Franco A, Vaccaro M. Recent trends in the development of heat exchangers for geothermal systems. 35th UIT Heat Transfer Conference (UIT2017). *IOP Conf. Series: Journal of Physics*, 2017; 923: 012044.
- [19] Galgaro A, Farina Z, Emmi G, de Carli M, Blum P. Feasibility analysis of a borehole heat exchanger (BHE) array to be installed in high geothermal fluid area: The case of the Euganean thermal Basin, Italy. *Renewable Energy*, 2015; 78: 93-104.
- [20] Hakki Aydin, Sukru Merey Potential of geothermal energy production from depleted gas fields: A case study of Dodan Field, Turkey. *Renewable Energy*, 164, 2020. 1076-1088, <https://doi.org/10.1016/j.renene.2020.10.057>
- [21] Fyk M, Biletskyi V, Fyk I, Bondarenko V, Al-Sultan M. Improvement of an engineering procedure for calculating the non-isothermal transportation of a gas-liquid mixture. *Eastern-European Journal of Enterprise Technologies*, 2019; 5(99): 51-60.

- [22] Biletskyi V, Shendrik T, Sergeev P. Derivatography as the method of water structure studying on solid mineral surface. *Geomechanical Processes During Underground Mining*, 2012;.181-184.
- [23] Zhu J, Hu K, Lu X, Huang X, Liu K, Wu X. A review of geothermal energy resources, development, and applications in China: Current status and prospects. *Energy*, 2015; 93: 466-483.
- [24] Soltani M, Moradi Kashkooli F, Dehghani-Saniij AR, Nokhosteen A, Ahmadi-Joughi A, Gharali K, Mahbaz SB, Dusseault MB. A comprehensive review of geothermal energy evolution and development. *International Journal of Green Energy*, 2019; 16(13): 971-1009,
- [25] Bodvarsson GS, Pruess K, Stefansson V, Bjornsson S, Ojiambo SB. Summary of modeling studies of the East Olkaria geothermal field, Kenya. *International symposium on geothermal energy*, 1985. Kailua Kona, HI, USA.

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